Orifice Equation
Derivation

Orifices are an integral part of all fluid systems. For example, a variable flow valve is simply an orifice with a variable area. Often, simple fixed orifices are used in circuits to regulate flow or provide drain reliefs. This paper will show how the orifice equation is used to calculate flow, pressure drop, or orifice area. Flow through a simple orifice fully converts available potential energy to kinetic energy, so fluid *viscosity* has no effect.

Derivation [1]

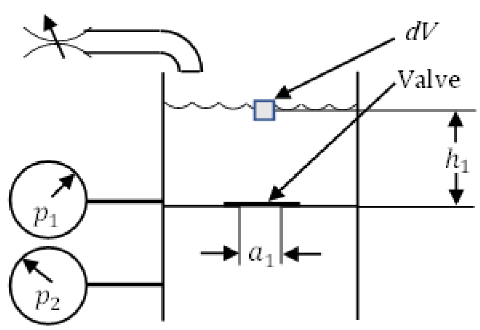

Figure 1

Consider Figure 1., where a tank is filled with a fluid to level h_1, acting under gravity, g, at standard atmospheric pressure, p_2. The fluid has a density ρ, where $\rho = m/V$ (*mass/volume*). The *absolute pressure*, p_1, at the bottom of the tank will be:

$$p_1 = \rho * g * h_1 + p_2 \qquad (F/L^2) \quad [1]$$

An opening, with area a_1, in the bottom of the tank is a valve and is closed with a cover.

In Figure 2, the valve is completely open to allow flow due to pressure $p_1$above atmospheric pressure p_2. If flow into the tank is regulated to maintain the fluid level at h_1, the fluid will flow through a_1 (L^2) to reach a maximum speed v (L/t) and smallest area a_2 (L^2) at a distance h_2 (d)

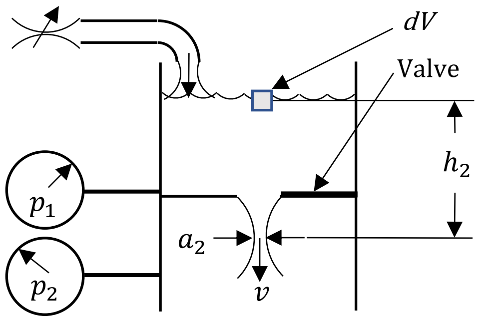

Figure 2

from the surface. This point is called the *vena contracta,* where available fluid potential energy is fully converted to the maximum kinetic energy.

Physics tells us the relationship where potential energy for a small volume of fluid *dV* (L^3), with mass dm, (m) falling through the distance h_2, for conversion to kinetic energy is:

$$^1/_2\, mv^2 = mgh \qquad [2]$$

The mass cancels and the equation is rearranged to:

$$v = \sqrt{2gh_2} \qquad [3]$$

The height h_2 and area a_2 are not known but can be accurately replaced by h_1 and area a_1 by using a coefficient, c_d, commonly called the discharge coefficient. The discharge coefficient is the ratio of measured actual flow divided by calculated expected flow and is known for many common orifices. For common fluid hydraulics use, the orifice is round with square edges. This type is reasonably accurate and easily manufactured. In common use, a value of c_d = 0.61 or c_d = 0.62 is used. Other types may be found in special applications and are sometimes checked in a measured flow to ensure accuracy.

Eq. [3] can be made useful by using a discharge coefficient to use only h_1 and a_1 and incorporate eq. [1] into eq. [3],

$^{\Delta p}/_{\rho} = \; g * h_1$ (where $\Delta p = p_1 - p_2$), we have:

$$v = C_f\sqrt{^{2\Delta p}/_{\rho}} \qquad \text{(L/t)} \quad [4]$$

Multiplying both sides of eq. [4] by area a_1 gives us the familiar equation for flow:

$$Q = C_d\, a\sqrt{2\Delta p/\rho} \qquad (L^3/t)\ \ [5]$$

This equation applies to any fluid flow through a simple orifice.
Consider, for example, Figure 3 where $\Delta p = p_1 - p_2$, as above.

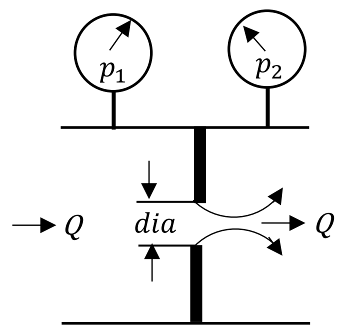

Figure 3

Of five variables in Eq. [5], $Q, c_d, a, \Delta p, \rho$, if four are known, the fifth can be calculated.

Orifice Example Part 1 shows how to calculate an orifice area for a dynamic braking application.

Orifice Calculator can help find values from the Orifice Equation.

Notes ---

1. See Torricelli's Law in Wikipedia for a good historical description.

Viscosity – A fluid property where energy is required to overcome the fluid's internal friction.

Absolute Pressure – Pressure measured above a perfect vacuum. For example, standard atmospheric pressure is 14.7 psi absolute pressure. Gage Pressure is referenced to atmospheric pressure. That is, 0 psi gage pressure = 14.7 psi absolute pressure.